Transverse fluctuations in the driven lattice gas

2003 J. Phys. A: Math. Gen. 36 L315

(http://iopscience.iop.org/0305-4470/36/21/101)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **36** (2003) L315–L320 PII: S0305-4470(03)62159-1

LETTER TO THE EDITOR

Transverse fluctuations in the driven lattice gas

Sergio Caracciolo[1], Andrea Gambassi[2,3], Massimiliano Gubinelli[4] and Andrea Pelissetto[5]

[1] Dipartimento di Fisica and INFN, Università degli Studi di Milano, Via Celoria 16, I-20133 Milano, and NEST-INFM, Italy
[2] Max-Planck-Institut für Metallforschung, Heisenbergstr. 3, D-70569 Stuttgart, Germany
[3] Institut für Theoretische und Angewandte Physik, Universität Stuttgart, Pfaffenwaldring 57, D-70569 Stuttgart, Germany
[4] Dipartimento di Matematica Applicata and INFN, Sez. di Pisa, Università degli Studi di Pisa, I-56100 Pisa, Italy
[5] Dipartimento di Fisica and INFN, Sez. di Roma I, Università degli Studi di Roma 'La Sapienza', I-00185 Roma, Italy

E-mail: Sergio.Caracciolo@mi.infn.it, gambassi@mf.mpg.de, m.gubinelli@dma.unipi.it and Andrea.Pelissetto@roma1.infn.it

Received 16 April 2003
Published 13 May 2003
Online at stacks.iop.org/JPhysA/36/L315

Abstract

We define a transverse correlation length suitable to discuss the finite-size scaling behaviour of an out-of-equilibrium lattice gas, whose correlation functions decay algebraically with the distance. By numerical simulations we verify that this definition has a good infinite-volume limit independent of the lattice geometry. We study the transverse fluctuations as they can select the correct field-theoretical description. By means of a careful finite-size scaling analysis, without tunable parameters, we show that they are Gaussian, in agreement with the predictions of the field theory proposed by Janssen, Schmittmann, Leung and Cardy.

PACS numbers: 05.50.+q, 05.70.Jk, 05.70.Ln

While the statistical mechanics of systems in thermal equilibrium is well established, no sound framework is available for nonequilibrium systems, although some interesting results have recently been obtained [1]. At present, most of the research focuses on very specific models. Among them, the driven lattice gas (DLG), introduced by Katz *et al* [2], has attracted much attention since it is one of the simplest nontrivial models with a nonequilibrium steady state [3]. The DLG is a generalization of the lattice gas. One considers a hypercubic lattice and for each site x introduces an occupation variable n_x, which can be either zero (empty site) or one (occupied site). Then, one introduces an external field E along a lattice direction and a generalization of the Kawasaki dynamics for the lattice gas with nearest-neighbour interactions. In practice, one randomly chooses a lattice link $\langle xy \rangle$, and, if $n_x \neq n_y$, proposes a particle jump which is accepted with Metropolis probabilities $w(\beta\Delta H + \beta E\ell)$, where $\ell = (1, 0, -1)$

for jumps (along, transverse, opposite) to the field direction, $w(x) = \min(1,\ e^{-x})$, and ΔH is the variation of the standard lattice-gas nearest-neighbour interaction

$$H = -4 \sum_{\langle xy \rangle} n_x n_y. \tag{1}$$

As usual, the parameter β plays the role of an inverse temperature. A nontrivial dynamics is obtained by considering periodic boundary conditions in the field direction. Indeed, in this case a particle current sets in, giving rise to a nonequilibrium stationary state that is non-Gibbsian.

At half filling, the DLG undergoes a second-order phase transition. Indeed, at high temperatures the steady state is disordered while at low temperatures the system is ordered: the particles condense forming a strip parallel to the field direction. The two temperature regions are separated by a phase transition occurring at the critical value $\beta_c(E)$ depending on the field E. $\beta_c(E)$ converges to the Ising critical value β_I for $E \to 0$, and, interestingly enough, increases with E, and saturates at a finite value $\beta_c(\infty)$ when E diverges. Such a transition is different in nature from the order/disorder one occurring in the lattice gas. For instance, it is *strongly anisotropic*, i.e. fluctuations in density correlation functions behave in a qualitatively different way depending on whether one considers points that belong to lines that are parallel or orthogonal to the field E.

Some time ago Janssen, Schmittmann, Leung and Cardy [4] (JSLC) proposed a Langevin equation for the coarse-grained density field, which incorporates the main features of the DLG, i.e. a conserved dynamics and the anisotropy induced by the external field, and should therefore describe the critical behaviour of the DLG. By means of a standard renormalization-group (RG) analysis, critical exponents were exactly computed in generic dimension d for $2 < d < d_c = 5$. This theory predicts a strongly anisotropic behaviour with correlations that increase with different exponents $\nu_\parallel$ and $\nu_\perp$ in the directions parallel and orthogonal to the external field [3]. The corresponding anisotropy exponent $\Delta = (\nu_\parallel/\nu_\perp) - 1$ can be exactly computed, and it comes out to be $\Delta = (8 - d)/3$. Another remarkable property of the JSLC theory is that transverse fluctuations (i.e., those associated with transverse wave vectors) become Gaussian in the critical limit. Extensive numerical simulations [5] have confirmed many predictions of the JSLC theory, although some notable discrepancies still remain. The conclusions of JSLC have been recently questioned by Garrido *et al* [6]. They argued that the DLG for $E = \infty$ is not described by the JSLC theory but is rather in the same universality class as the randomly driven lattice gas (RDLG). In this model, the direction of the external field is chosen randomly at each time step, so that the global Ising symmetry is restored. The RG analysis done in [7] leads to several notable differences. First, the upper critical dimension is $d_c = 3$. Second, $\Delta = 1 + O((3-d)^2)$, is well approximated by $\Delta = 1$ in $d = 2$ (the case we will consider in our numerical simulations). Finally, the transverse critical fluctuations are not Gaussian. Clearly, this universality class is quite different from the JSLC class. Nevertheless, some recent studies [8] claim that the $d = 2$ DLG belongs to the RDLG class.

In view of these contradictory results, new numerical investigations are necessary, in order to decide which of these two theories really describes the DLG universality class. In this paper, we will focus on the transverse fluctuations for the very simple reason that in the JSLC theory there is a very strong prediction: transverse correlation functions are Gaussian. Therefore, besides critical exponents, one can compute scaling functions of several observables *exactly* and construct unambiguous tests against data.

A stringent way to test these JSLC predictions is to investigate the finite-size scaling (FSS) behaviour of several observables. For an isotropic model on a lattice L^d, the FSS limit corresponds to $t \equiv 1 - \beta/\beta_c \to 0$, $L \to \infty$, keeping $tL^{1/\nu}$ constant. This has to be modified for strongly anisotropic models [9] such as the DLG. It has been argued that, for a geometry $L_\parallel \times L_\perp^{d-1}$, one has to keep fixed both combinations $tL_\parallel^{1/\nu_\parallel}$ and $tL_\perp^{1/\nu_\perp}$, and therefore also the so-called aspect ratio $S_\Delta = L_\parallel^{1/(1+\Delta)}/L_\perp$. Then, if an observable $\mathcal{O}$ diverges at criticality as $t^{-z_\mathcal{O}}$, in a finite lattice one has

$$\mathcal{O}(\beta; L_\parallel, L) \approx L^{z_\mathcal{O}/\nu_\perp} f_\mathcal{O}(t^{-\nu_\perp}/L, S_\Delta) \tag{2}$$

where we have neglected subleading scaling corrections. To simplify the notation we write L instead of $L_\perp$ and, below, ξ_L for the *transverse* correlation length. In many numerical studies equation (2) has been tested for several observables, but this can be a very weak test since several parameters, β_c, $\nu_\perp$, $z_\mathcal{O}$ and Δ, must be tuned in order to fit the numerical data. A stronger FSS test can be performed if one uses a suitably defined correlation length. In this case, using a transverse (infinite-volume) correlation length ξ_∞, equation (2) may be written in the form

$$\mathcal{O}(\beta; L_\parallel, L) \approx L^{z_\mathcal{O}/\nu_\perp} \tilde{f}_\mathcal{O}(\xi_\infty(\beta)/L, S_\Delta). \tag{3}$$

In this equation β_c does not explicitly appear and $z_\mathcal{O}$ and $\nu_\perp$ enter only through their ratio, cancelling when one looks directly at the correlation length. One can also eliminate this unknown by considering the ratio of the observables on two different lattices with transverse sizes L and αL. In this case we have

$$\frac{\mathcal{O}(\beta; \alpha^{1+\Delta} L_\parallel, \alpha L)}{\mathcal{O}(\beta; L_\parallel, L)} \approx F_\mathcal{O}\left(\frac{\xi(\beta; L_\parallel, L)}{L}, S_\Delta, \alpha\right) \tag{4}$$

where only Δ has to be fixed *a priori*.

In order to test equations (3) and (4), one has to define a finite-size correlation length. In the DLG this is not obvious because correlation functions always decay algebraically at large distances [10]. A parallel correlation length was defined in [11], but it suffers from many ambiguities [3]. The definition of a transverse correlation length is even more difficult, because of the presence of negative correlations at large distances [3, 5].

In this paper we propose a new definition that generalizes the second-moment correlation length used in equilibrium systems. The basic observation is that the infinite-volume *wall–wall* correlation function decays exponentially (i.e., $\left\langle \sum_{x_\parallel} n_x \sum_{y_\parallel} n_y \right\rangle_{\text{conn}} \to \mathrm{e}^{-\kappa|x_\perp - y_\perp|}$), so that a transverse correlation length can be naturally defined in the thermodynamic limit. The extension of this definition to finite volumes requires some care because of the conserved dynamics, which makes the two-point function vanish at zero momentum. Here, we will use the results of [12]. Given the Fourier transform $\tilde{G}(q)$ of the two-point correlation function $\langle n_x n_0 \rangle$, we focus on the transverse correlation $\tilde{G}_\perp(q) \equiv \tilde{G}(\{q_\parallel = 0, q_\perp = q\})$ and define

$$\xi_{ij} \equiv \sqrt{\frac{1}{\hat{q}_j^2 - \hat{q}_i^2}\left(\frac{\tilde{G}_\perp(q_i)}{\tilde{G}_\perp(q_j)} - 1\right)} \tag{5}$$

where $\hat{q}_n = 2\sin(\pi n/L)$ is the lattice momentum. If the infinite-volume transverse wall–wall correlation function decays exponentially, $\tilde{G}_\perp(q)$ has a regular expansion in powers of q^2 and

$$\tilde{G}_\perp^{-1}(q) \approx \chi_\infty^{-1}[1 + b^2 q^2 + O(q^4)] \tag{6}$$

where the coefficient b of q^2 naturally defines a transverse correlation length ξ_∞. We expect equation (6) to hold also in a finite box. Then, starting from equation (5), it is easy to show that ξ_{ij} converges to ξ_∞ as $L \to \infty$, justifying our definition of finite-volume correlation length. In the subsequent analysis we consider ξ_{13} as the finite-volume (transverse) correlation length $\xi_L(T) \equiv \xi\left(T; S_\Delta^{1+\Delta} L^{1+\Delta}, L\right)$. As in previous studies, we also define a finite-volume transverse susceptibility as $\chi_L = \tilde{G}_\perp(2\pi/L)$.

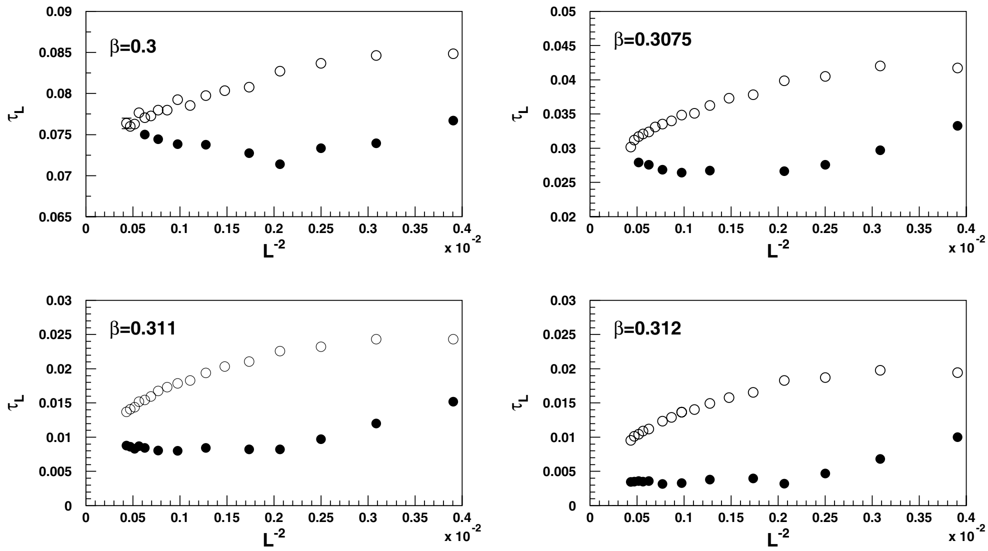


Figure 1. τ_L for different inverse temperatures β. Filled (respectively open) circles refer to geometries with aspect ratio S_2 (respectively S_1) fixed. Here $S_2 \approx 0.200$, $S_1 \approx 0.106$. Errors are smaller than the size of the points.

As we already said, in the JSLC theory transverse fluctuations are Gaussian. This allows us to compute the scaling function appearing in equation (4) for ξ_L and χ_L. If $\tilde{G}_\perp(q)$ is Gaussian we have

$$F_\xi(z, S_2, \alpha) = [1 - (1 - \alpha^{-2})(2\pi)^2 z^2]^{-1/2} \tag{7}$$

with $z \equiv \xi_L/L$ and $F_\chi(z, S_2, \alpha) = F_\xi(z, S_2, \alpha)^2$. We can also compute $\tilde{f}_\xi(x, S_2)$, see equation (3), obtaining

$$\tilde{f}_\xi(x, S_2) = (4\pi^2 + 1/x^2)^{-1/2}. \tag{8}$$

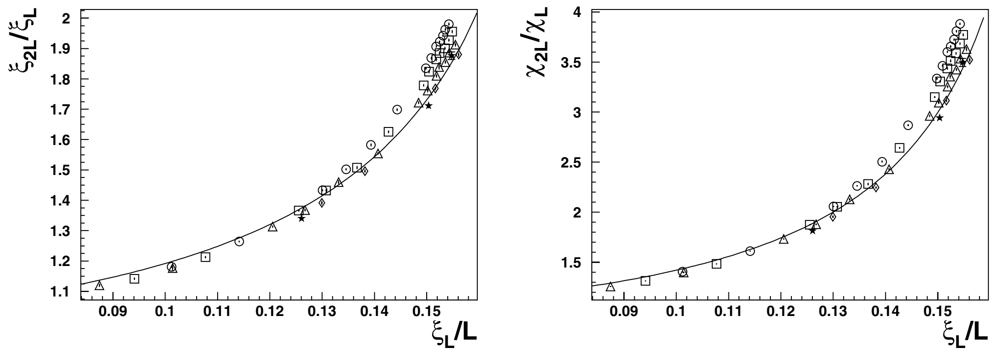
In this paper we want to make a high-precision test of the theoretical predictions of the JSLC theory, equations (7) and (8). We work in two dimensions at infinite driving field. Since we wish to test the JSLC predictions we fix $\Delta = 2$ and consider lattice sizes with $S_2 \approx 0.200$. The largest lattice corresponds to $L_\parallel = 884$, $L = 48$. For each lattice size, we compute χ_L and ξ_L for several values of β lying between 0.28 and 0.312.

As a preliminary test, we verify that our definition of ξ_L has a good thermodynamic limit. For this purpose and reasons presented below, we introduce the following quantity:

$$\tau_L(\beta) \equiv \xi_L^{-2}(\beta) - 4\pi^2 L^{-2}. \tag{9}$$

In figure 1 we plot $\tau_L(\beta)$ versus $1/L^2$ at several inverse temperatures β. For each β, $\tau_L(\beta)$ converges to a finite constant, showing that our definition has a finite infinite-volume limit. Moreover, the same result is obtained by using sequences of lattices with S_2 or S_1 fixed: the result does not depend on the way in which $L_\parallel$ and L go to infinity. As expected, when the temperature approaches the critical value, it is necessary to use larger and larger lattices to see the convergence to the infinite-volume limit. For lattices with S_2 fixed we observed an intermediate region of values of L in which τ_L is apparently constant. Such a region widens as β approaches the critical point and is therefore in excellent agreement with the relation

$$\xi_\infty^{-2}(\beta) \approx \tau_L(\beta) = \xi_L^{-2}(\beta) - 4\pi^2 L^{-2} \tag{10}$$



(*a*) (*b*)

Figure 2. FSS curve for (*a*) the transverse correlation length and (*b*) the transverse susceptibility at fixed $S_2 \approx 0.200$. Different symbols correspond to different lattice sizes: $L = 16\,(\circ), 18\,(\Box), 20\,(\triangle), 22\,(\Diamond), 24\,(\bigstar)$. The solid curve in (*a*) is the function $F_\xi(z, S_2, 2)$ defined in equation (7), while in (*b*) it is the function $F_\chi(z, S_2, 2) = F_\xi(z, S_2, 2)^2$.

in the FSS limit $L \to \infty, \beta \to \beta_c$. Note that the corrections beyond the one shown here will depend in general on the chosen value of S_2 and are particularly small for our choice $S_2 \approx 0.200$. Equation (10) immediately gives equation (8). Thus, the results presented in figure 1 are perfectly consistent with $\Delta = 2$ and the JSLC prediction for $\tilde{f}_\xi(x, S_2)$.

We want now to make a precise test of equation (7). In figure 2(*a*) we report the results of our simulations for the ratio ξ_{2L}/ξ_L as a function of ξ_L/L for lattice sizes with fixed S_2, and we compare them with equation (7). We stress that the theoretical curve is not a fit to the data: there is no free parameter to be chosen! Though the agreement is not perfect, we note that the points closer to the theoretical curve correspond to larger lattices.

Using the universal function $F_\xi(z, S_2, 2)$, we can extrapolate our data to infinite volume using the general strategy of [13]. Correspondingly, we obtain $\beta_c = 0.312\,694(18)$ and verify that, for small t, $\xi_\infty \sim t^{-1/2}$ as predicted by JSLC.

We can perform the same test for the susceptibility. In figure 2(*b*) we report our numerical results for χ_L together with the theoretical JSLC prediction. We observe a good agreement between theory and Monte Carlo results.

We have also measured the transverse Binder cumulant g_L [3], observing that $g_L(\beta_c) \sim L^{-0.4(2)}$ for $L \to \infty$. Thus, the Binder cumulant vanishes at the critical point, again in agreement with the idea that transverse fluctuations are Gaussian. The small power with which $g_L(\beta_c)$ vanishes hints at the presence of logarithmic corrections. Further details of our analysis will be presented elsewhere [14].

To summarize, we carried out extensive Monte Carlo simulations on the DLG and performed an incisive FSS analysis. We introduced a new and unambiguous measure of the transverse correlation length, as well as a new scaling function. We determined not only exponents, but also compared the measured scaling function to the theoretical one with *no fit parameters*. Our conclusion is that the critical behaviour of transverse fluctuations is indeed Gaussian, in perfect agreement with the theoretical predictions of JSLC.

Acknowledgments

It is a pleasure to thank B Schmittmann and R K P Zia for useful discussions, and M Muñoz for his attention and correspondence. We acknowledge the support of the INFM through the

Advanced Parallel Computing Project at CINECA. These computations were carried out in part on the Computational Physics Cluster at New York University, which was supported by NSF grant PHY-0116590.

References

[1] Derrida B, Lebowitz J L and Speer E R 2002 *Phys. Rev. Lett.* **89** 030601
Bertini L, De Sole A, Gabrielli D, Jona-Lasinio G and Ladim C 2002 *J. Stat. Phys.* **107** 635
For an overview, see Ruelle D 2001 *Nature* **411** 27

[2] Katz S, Lebowitz J L and Spohn H 1983 *Phys. Rev.* B **28** 1655
Katz S, Lebowitz J L and Spohn H 1984 *J. Stat. Phys.* **34** 497

[3] Schmittmann B and Zia R K P 1995 Statistical mechanics of driven diffusive systems *Phase Transitions and Critical Phenomena* vol 17 ed C Domb and J L Lebowitz (London: Academic)
Marro J and Dickman R 1999 *Nonequilibrium Phase Transitions in Lattice Models* (Cambridge: Cambridge University Press)

[4] Janssen H K and Schmittmann B 1986 *Z. Phys.* B **64** 503
Leung K-t and Cardy J L 1986 *J. Stat. Phys.* **44** 567
Leung K-t and Cardy J L 1986 *J. Stat. Phys.* **45** 1087 (erratum)

[5] Zhang M Q, Wang J-S, Lebowitz J L and Vallés J L 1988 *J. Stat. Phys.* **52** 1461
Leung K-t 1991 *Phys. Rev. Lett.* **66** 453
Leung K-t 1992 *Int. J. Mod. Phys.* C **3** 367
Wang J S 1996 *J. Stat. Phys.* **82** 1409
Leung K-t and Wang J S 1999 *Int. J. Mod. Phys.* C **10** 853

[6] Garrido P L, de los Santos F and Muñoz M A 1998 *Phys. Rev.* E **57** 752
Garrido P L and de los Santos F 1999 *J. Stat. Phys.* **96** 303
Garrido P L, Muñoz M A and de los Santos F 2000 *Phys. Rev.* E **61** R4683

[7] Schmittmann B and Zia R K P 1991 *Phys. Rev. Lett.* **66** 357
Schmittmann B 1993 *Europhys. Lett.* **24** 109

[8] Achahbar A, Garrido P L, Marro J and Muñoz M A 2001 *Phys. Rev. Lett.* **87** 195702
Albano E V and Saracco G 2002 *Phys. Rev. Lett.* **88** 145701

[9] Binder K and Wang J S 1989 *J. Stat. Phys.* **55** 87

[10] Grinstein G 1991 *J. Appl. Phys.* **69** 5441

[11] Vallés J L and Marro J 1986 *J. Stat. Phys.* **43** 441
Vallés J L and Marro J 1987 *J. Stat. Phys.* **49** 89

[12] Caracciolo S, Gambassi A, Gubinelli M and Pelissetto A 2001 *Eur. Phys. J.* B **20** 255

[13] Caracciolo S, Edwards R G, Ferreira S J, Pelissetto A and Sokal A D 1995 *Phys. Rev. Lett.* **74** 2969
Caracciolo S, Edwards R G, Pelissetto A and Sokal A D 1995 *Phys. Rev. Lett.* **75** 1891

[14] Caracciolo S, Gambassi A, Gubinelli M and Pelissetto A in preparation